

**THEMATICS**  
**ANDARD LEVEL**  
**PER 2**

CIMEN

our 30 minutes

Candidate session number

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Examination code

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**TRUCTIONS TO CANDIDATES**

Write your session number in the boxes above.

Do not open this examination paper until instructed to do so.

A graphic display calculator is required for this paper.

Section A: answer all questions in the boxes provided.

Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.

At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.

A clean copy of the **Mathematics SL formula booklet** is required for this paper.

The maximum mark for this examination paper is [90 marks].

**SECTION A (47 Marks)**

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

In an arithmetic series, the first term is  $-7$  and the sum of the first 20 terms is 620.

- (a) Find the common difference. [3 marks]
- (b) Find the value of the 78<sup>th</sup> term. [2 marks]

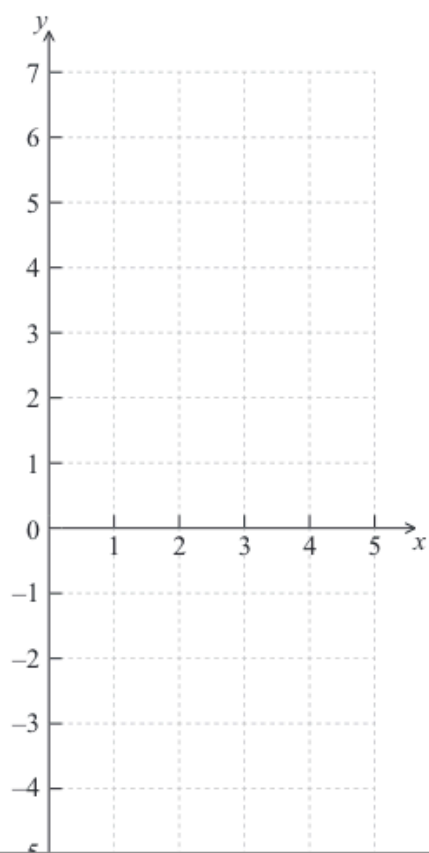
[illegible]

2. [Maximum mark: 7]

Let  $f(x) = 4x - e^{x-2} - 3$ , for  $0 \leq x \leq 5$ .

(a) Find the  $x$ -intercepts of the graph of  $f$ . [3 marks]

(b) On the grid below, sketch the graph of  $f$ . [3 marks]



(c) Write down the gradient of the graph of  $f$  at  $x = 3$ .

[1 mark]

.....  
.....  
.....

[illegible]

[illegible]

5. [Maximum mark: 7]

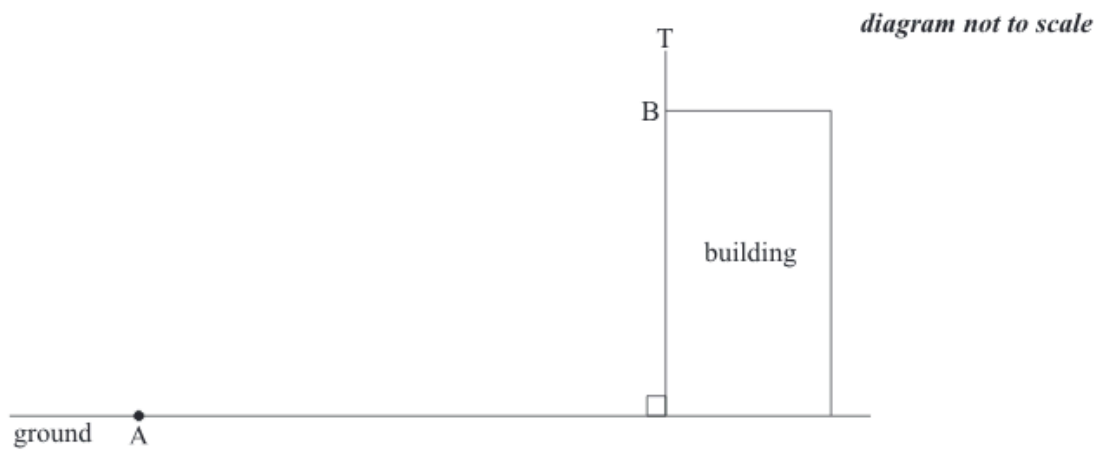
The probability of obtaining heads on a biased coin is 0.4. The coin is tossed 600 times.

- (a) (i) Write down the mean number of heads.
- (ii) Find the standard deviation of the number of heads. [4 marks]
- (b) Find the probability that the number of heads obtained is less than one standard deviation away from the mean. [3 marks]

[illegible]

6. [Maximum mark: 7]

The following diagram shows a pole BT 1.6 m tall on the roof of a vertical building. The angle of depression from T to a point A on the horizontal ground is  $35^\circ$ . The angle of elevation of the top of the building from A is  $30^\circ$ .



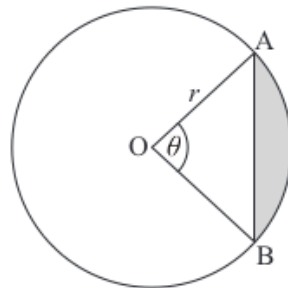
Find the height of the building.

[illegible]



7. [Maximum mark: 8]

A circle centre O and radius  $r$  is shown below. The chord [AB] divides the area of the circle into two parts. Angle AOB is  $\theta$ .



- (a) Find an expression for the area of the shaded region. [3 marks]
- (b) The chord  $[AB]$  divides the area of the circle in the ratio 1:7. Find the value of  $\theta$ . [5 marks]

[illegible]

Do **NOT** write solutions on this page.

### SECTION B (43 Marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 13]

Each day, a factory recorded the number ( $x$ ) of boxes it produces and the total production cost ( $y$ ) dollars. The results for nine days are shown in the following table.

$x$	26	44	65	43	50	31	68	46	57
$y$	400	582	784	625	699	448	870	537	724

(a) Write down the equation of the regression line of  $y$  on  $x$ .

[2 marks]

Use your regression line as a model to answer the following.

(b) Interpret the meaning of

(i) the gradient;

(ii) the  $y$ -intercept.

[2 marks]

(c) Estimate the cost of producing 60 boxes.

[2 marks]

(d) The factory sells the boxes for \$19.99 each. Find the least number of boxes that the factory should produce in one day in order to make a profit.

[3 marks]

(e) Comment on the appropriateness of using your model to

(i) estimate the cost of producing 5000 boxes;

(ii) estimate the number of boxes produced when the total production cost is \$540.

[4 marks]

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

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Do **NOT** write solutions on this page.

9. [Maximum mark: 16]

Let  $h(x) = \frac{2x-1}{x+1}$ ,  $x \neq -1$ .

(a) Find  $h^{-1}(x)$ . [4 marks]

(b) (i) Sketch the graph of  $h$  for  $-4 \leq x \leq 4$  and  $-5 \leq y \leq 8$ , including any asymptotes.

(ii) Write down the equations of the asymptotes.

(iii) Write down the  $x$ -intercept of the graph of  $h$ . [7 marks]

(c) Let  $R$  be the region in the first quadrant enclosed by the graph of  $h$ , the  $x$ -axis and the line  $x = 3$ .

(i) Find the area of  $R$ .

(ii) Write down an expression for the volume obtained when  $R$  is revolved through  $360^\circ$  about the  $x$ -axis. [5 marks]

## 10. [Maximum mark: 14]

A rock falls off the top of a cliff. Let  $h$  be its height above ground in metres, after  $t$  seconds.

The table below gives values of  $h$  and  $t$ .

$t$ (seconds)	1	2	3	4	5
$h$ (metres)	105	98	84	60	26

- (a) Jane thinks that the function  $f(t) = -0.25t^3 - 2.32t^2 + 1.93t + 106$  is a suitable model for the data. Use Jane's model to
- (i) write down the height of the cliff;
  - (ii) find the height of the rock after 4.5 seconds;
  - (iii) find after how many seconds the height of the rock is 30 m. [5 marks]
- (b) Kevin thinks that the function  $g(t) = -5.2t^2 + 9.5t + 100$  is a better model for the data. Use Kevin's model to find when the rock hits the ground. [3 marks]
- (c) (i) On graph paper, using a scale of 1 cm to 1 second, and 1 cm to 10 m, plot the data given in the table.
- (ii) By comparing the graphs of  $f$  and  $g$  with the plotted data, explain which function is a better model for the height of the falling rock. [6 marks]
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